

ON ARCS AND Ω

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The gravitational lens effect of galaxy clusters can produce large arcs from source galaxies in their background. Typical source redshifts of ~ 1 require clusters at $z \sim 0.3$ for arcs to form efficiently. Given the cluster abundance at the present epoch, the fewer clusters exist at $z \sim 0.3$ the higher Ω_0 is, because the formation epoch of galaxy clusters strongly depends on Ω_0 . In addition, at fixed Ω_0 , clusters are less concentrated, and hence less efficient lenses, when the cosmological constant is positive, $\Omega_\Lambda > 0$. Numerical cluster simulations show that the expected number of arcs on the sky is indeed a sensitive function of Ω_0 and Ω_Λ . The numerical results are compatible with the statistics of observed arcs only in a universe with low matter density, $\Omega_0 \sim 0.3$, and zero cosmological constant. Other models fail by one or two orders of magnitude, rendering arc statistics a sensitive probe for cosmological parameters.

1 Introduction

Given the average mass of a rich galaxy cluster, M_c , and the spatial number density of such clusters in our neighbourhood, n_c , we can find the fraction of cosmic material contained in rich clusters today,

$$F'_c = \frac{M_c n_c}{\rho_{\text{cr}} \Omega_0} \approx \frac{9 \times 10^{-3}}{\Omega_0}, \quad (1)$$

where ρ_{cr} is the critical cosmic matter density, and Ω_0 is the usual density parameter. The *ansatz* by Press & Schechter (1974) asserts that the fraction of cosmic material accumulated by clusters at redshift z is

$$F_c(z) = \frac{1}{2} \text{erfc} \left(\frac{\delta_c}{\sqrt{2} \sigma_c \mathcal{D}(z)} \right), \quad (2)$$

where σ_c is the *rms* density fluctuation on the linear cluster scale today, and $\mathcal{D}(z)$ is the growth factor for cosmic structures, normalized to $\mathcal{D}(0) = 1$. δ_c is the linear density contrast of a spherical top-hat perturbation at collapse time, and $\text{erfc}(x)$ is the complementary error function. Demanding $F_c(0) = F'_c$, σ_c is fixed to the local cluster abundance. The evolution of $F_c(z)$ with redshift then depends on cosmology because the growth factor $\mathcal{D}(z)$ does. This leads

to the well-known results that (i) clusters form late in cosmic history, and (ii) cluster formation is significantly delayed in high-density compared to low-density universes (e.g. N. Bahcall, these proceedings).

The ability of a galaxy cluster to act as a strong gravitational lens (i.e., to produce arcs) depends on the geometry of the lens system. Let $D_{\text{eff}} = D_d D_{ds} D_s^{-1}$ be the effective lens distance, with $D_{d,ds,s}$ the angular-diameter distances between observer and lens, lens and source, and observer and source, respectively. D_{eff} is a measure for the lensing efficiency of a given mass distribution. D_{eff} peaks at $z \sim 0.2 - 0.3$ for sources at a typical redshift $z_s \sim 1$, quite independent of cosmology.

It follows that an efficient formation of arcs requires that there be sufficiently many clusters in place and compact enough at redshifts $z \sim 0.2 - 0.3$. This establishes the link between arc statistics and cosmology. Quite obviously, the number of efficient cluster lenses per unit redshift is estimated by

$$\frac{dN_{\text{lens}}}{dz} \propto F_c(z) \times (1+z)^3 \times D_{\text{eff}}^2 \times \left| \frac{dV(z)}{dz} \right|, \quad (3)$$

where the square on D_{eff} approximates the dependence of the lensing cross section on D_{eff} , and $dV(z)$ is the proper cosmic volume of a spherical shell of radius z and width dz . Figure 1 illustrates dN_{lens}/dz as a function of redshift for $\Omega_0 = 1$ and $\Omega_0 = 0.3$, both for $\Omega_\Lambda = 0$. Evidently, there is a huge difference of about two orders of magnitude, clusters in the low-density universe being much more efficient in producing arcs than in the Einstein-de Sitter universe. This straightforward argument leads one to expect that the number of observed arcs could be a sensitive discriminator between cosmological models.

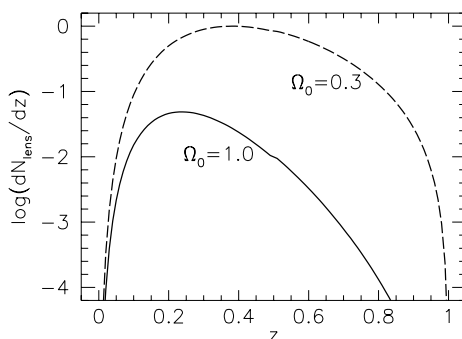


Fig. 1 — Estimate of the number of lenses per unit redshift, dN_{lens}/dz , as defined in the text, for high- and low-density universes. Note the logarithmic scale of the ordinate. The curves illustrate that the number of efficient lenses is expected to be lower by about two orders of magnitude in high- compared to the low-density model.

2 Simulations

We use numerical simulations to quantify the expected effect more precisely. Clusters are taken from four different cosmological simulations, the parameters of which are summarized in Tab. 1. The models indexed by 1 were kindly supplied by the GIF collaboration (cf. S.D.M. White or J.M. Colberg, these proceedings), those indexed by 2 were simulated with a different numerical algorithm. In total, we use nine simulated clusters for (S, Λ ,O)CDM, and five for τ CDM.

| Model | Ω_0 | Ω_Λ | h | σ_8 | Γ |
|----------------|------------|------------------|-----|------------|----------|
| SCDM1 | 1.0 | 0.0 | 0.5 | 0.60 | 0.50 |
| τ CDM1 | 1.0 | 0.0 | 0.5 | 0.60 | 0.21 |
| Λ CDM1 | 0.3 | 0.7 | 0.7 | 0.90 | 0.21 |
| OCDM1 | 0.3 | 0.0 | 0.7 | 0.85 | 0.21 |
| SCDM2 | 1.0 | 0.0 | 0.5 | 0.60 | 0.50 |
| Λ CDM2 | 0.3 | 0.7 | 0.7 | 1.12 | 0.21 |
| OCDM2 | 0.3 | 0.0 | 0.7 | 1.12 | 0.21 |

Tab. 1 — Summary of the parameters used for the cluster simulations. h is the Hubble constant in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$, Γ is the shape parameter of the power spectrum, and the other parameters have their conventional meaning.

Each cluster is studied at ten time steps between redshifts 1 and 0, projecting it along each of the three independent spatial directions, ending up with roughly 10^3 lensing mass distributions. For each of these, the arc cross section is computed numerically, mapping elliptical sources at redshift $z_s = 1$ that are placed on an adaptive grid tracing the caustic curves of the lenses. In total, we classify all images of about 1.3×10^6 sources. This procedure yields arc cross sections as a function of cluster redshift, $\sigma(z)$, for the four cosmological models used. The arc optical depth, i.e., the probability for a source to be imaged as an arc with specified properties, is then given by a volume-weighted integral of $\sigma(z)$ over redshift, multiplied by the cluster number density n_c and divided by the area of the source sphere.

3 Results

The optical depth normalized by the cluster number density, $n_c^{-1}\tau$, is shown in Fig. 2 for the models (S, Λ ,O)CDM. The result for the τ CDM model is almost identical to that for the SCDM model, and is therefore omitted. In summary, the optical depth for large arcs, i.e. such with a length-to-width ratio ≥ 10 , is highest for the OCDM model and lowest for the SCDM model, with differences of about an order of magnitude between each of the models.

Combining the optical depth with the number densities of observed clusters and of appropriately bright sources, we find that the number of arcs on the

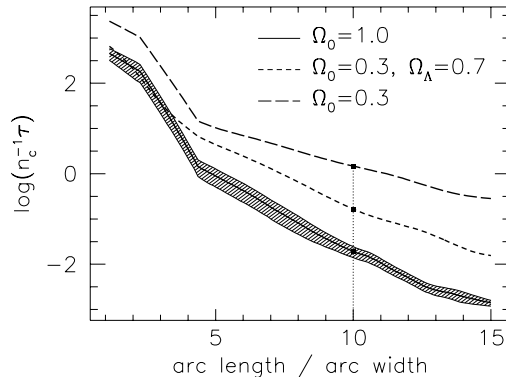


Fig. 2 — Optical depth $n_c^{-1}\tau$, as a function of the length-to-width ratio of the arcs. The hatched region centered on the SCDM curve (solid line) illustrates 1σ bootstrapping errors. The dots mark the optical depth for arcs with length-to-width ratio ≥ 10 . The optical depth is highest for the OCDM model, and lower by one order of magnitude each for Λ CDM and SCDM, respectively.

whole sky expected from our simulations is

$$N_{\text{arcs}} \sim \begin{cases} 2400 & \text{OCDM} \\ 280 & \Lambda\text{CDM} \\ 36 & \text{SCDM} \end{cases} . \quad (4)$$

The observed number of arcs, estimated from the EMSS arc survey and extrapolated to the whole sky, falls within $1500 - 2300$. We therefore conclude that *the only of our cosmological models for which the expected number of arcs comes near the observed number is the open CDM model*. The others fail by one or two orders of magnitude. This result can be understood by (i) the delayed cluster formation in high- Ω_0 universes, combined with lensing efficiency, and (ii) the higher concentration of clusters in low-density models without Ω_Λ compared to such with $\Omega_\Lambda > 0$, combined with the sensitivity of lensing to compactness. For details, see Bartelmann et al. (1997).

Acknowledgments

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